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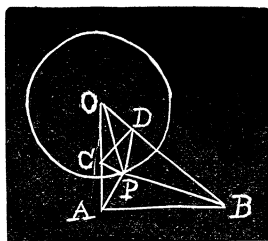
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BP make equal angles with the radius drawn to P .

This is known as Alhazen's Problem and does not admit of a solution by ruler and compasses only. However, by the use of an hyperbola an approximate solution can be effected.

Construction. Let O be the center of the circle, a its radius. Take C and D so that $AO \cdot OC = a^2 = BO \cdot OD$.

Now, the locus of the vertices of the triangles whose base is CD and whose base angles have a constant difference ($\angle OCD - \angle ODC$) is well known to be a hyperbola. This will cut the circle in four points, of which let P be one. This is the point required.

Proof. $\angle CDP - \angle DCP = \angle OCD - \angle ODC$ (by construction). Transposing and adding,

$$\therefore \angle OCP = \angle ODP \dots (1).$$

Also in the triangles AOP , POC , we have $AO:OP = OP:OC$ (by construction).

$$\therefore \angle APO = \angle OCP. \text{ Similarly, } \angle BPO = \angle ODP \text{ by (1).}$$

$$\therefore \angle APO = \angle BPO, \text{ and } \therefore \angle APR = \angle BPR. \quad \text{Q. E. F. Q. E. D.}$$

350. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Given the quadrilateral $AB=a=225$, $BC=b=153$, $CD=c=207$, $DA=d=135$, $AC=e=240$. Find the side of the square inscribed in this quadrilateral having a corner in each side.

Solution by the PROPOSER.

Let $ABCD$ be the given quadrilateral, $EFGH$ the inscribed square, $RPQS$ the circumscribed rectangle having its sides parallel to the sides of the square. Draw AIJ , BUT , CNL , DVM perpendicular, respectively, to EF and HG , FG and HE , HG and EF , HE and GF .

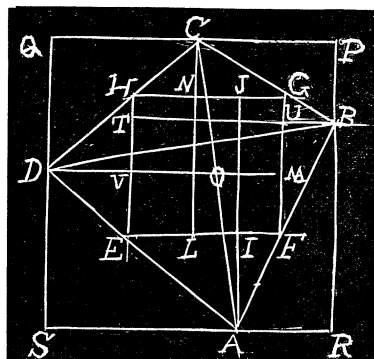
Let $AB=a$, $BC=b$, $CD=c$, $DA=d$, $AC=e$, $BD=f$, O the intersection of AC , BD , $\angle COB = \beta$, $\angle ACL = \theta = \angle CAJ$, $\angle BDM = \phi = \angle DBT$, $\angle BAC = \delta$, $\angle DAC = \gamma$, $\angle BCA = \rho$, $\angle DCA = \mu$, area $ABCD = \Delta$.

Then $\phi = \frac{1}{2}\pi - (\beta - \theta)$, $RB = a \cos(\delta - \theta)$, $RA = a \sin(\delta - \theta)$, $DS = d \cos(\gamma + \theta)$, $AS = d \sin(\gamma + \theta)$, $DQ = c \cos(\mu - \theta)$, $CQ = c \sin(\mu - \theta)$, $BP = b \cos(\rho + \theta)$, $PC = b \sin(\rho + \theta)$, $AI + x + CN = AJ + CN = e \cos \theta$, $BU + x + DV = BT + DV = f \cos \phi$.

$$\therefore BT + DV = f \sin(\beta - \theta).$$

$$x^2 + \frac{1}{2}x(AI + BU + CN + DV) = \Delta = \frac{1}{2}x(AI + x + CN) + \frac{1}{2}x(BU + x + DV).$$

$$\therefore x[ec \cos \theta + f \sin(\beta - \theta)] = 2\Delta \dots (1).$$



$$\Delta + \frac{1}{4}[a^2 \sin 2(\delta - \theta) + b^2 \sin 2(\rho + \theta) + c^2 \sin 2(\mu - \theta) + d^2 \sin 2(\gamma + \theta)] \\ = ef \cos \theta \sin(\beta - \theta) \dots (2).$$

Reducing (2), and remembering that $4\Delta - 2ef \sin \beta = 0$, we get

$$\tan 2\theta = \frac{a^2 \sin 2\delta + b^2 \sin 2\rho + c^2 \sin 2\mu + d^2 \sin 2\gamma - 2ef \sin \beta}{a^2 \cos 2\delta - b^2 \cos 2\rho + c^2 \cos 2\mu - d^2 \cos 2\gamma - 2ef \cos \beta} \dots (3).$$

When $a=225$, $b=153$, $c=207$, $d=135$, $e=240$. Then $f=277.4$, $\Delta=30656.46$, $\delta=38^\circ 14' 54''$, $\rho=65^\circ 33' 40''$, $\gamma=59^\circ 24' 36''$, $\mu=34^\circ 9' 22''$, $\beta=67^\circ 3' 52''$.

$$\therefore \tan 2\theta = -.345608 = 2 \tan \theta / (1 - \tan^2 \theta).$$

$$\therefore \tan \theta = 5.954833 \text{ or } -0.167930.$$

$$\theta = 80^\circ 28' 2'' \text{ or } 170^\circ 28' 2'' \text{ and } x = -2496.97 \text{ or } -121.044.$$

Therefore, two squares can be inscribed in this quadrilateral, the smaller one truly inscribed and the larger with its corners on the sides.

Also solved by J. Scheffer, and V. M. Spunar.

351. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Given an isosceles right triangle with hypotenuse h ; an isosceles triangle with two sides h and two angles $A=22^\circ 30'$; a right angle triangle with the same angle A and opposite side $h/\sqrt{2}$; a triangle with the same angle A , opposite side h , and an angle 45° . Form a triangle whose four pieces are these four triangles, and prove geometrically that it is isosceles.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let ABD be the isosceles right triangle with hypotenuse $BD=h$, sides AB , $AD=h/\sqrt{2}$.

Produce AB to R , making $BR=BD$ and connect RD . Since $\angle ABD=45^\circ$, $\angle RBD=135^\circ$.

$\therefore \angle BRD = \angle RDB = 22^\circ 30' = A$, and BRD is the isosceles triangle with sides h and opposite angles A . Take Q in DA produced so that $AQ=AD$ and join RQ . Then the right triangle ARQ has angle $ARQ=A$ and $AQ=h/\sqrt{2}$. Produce AD to P making $DP=h$ and join RP . Triangle DRP has angle $DRP=A$, $\angle P=45^\circ$, and side DP opposite $\angle DRP=h$.

Now $\angle PQR=67^\circ 30'=3A$, $\angle ARQ = \angle ARD = \angle DRP=A$.

$\therefore \angle QRP=3A$. $\therefore \angle Q = \angle QRP$, and the triangle PQR is isosceles.

